COMP 353 - ASSIGNMENT 2 - KEVIN MCALLISTER(40031326)

Armstrong's Axioms:

{Y ⊆ X} → {X → Y} // Reflexivity (Trivial FDs)

{X → Y} → {XZ → YZ ∀ Z} // Augmentation

{X → Y, Y → Z} → {X → Z} // Transitivity

{X → Y, X → Z} → {X → YZ} // Union Rule

{X → YZ} → {X → Y, X → Z} // Decomposition Rule

{X → Y, WY → Z} → {XW → Z} // Pseudotransitivity Rule

1.

a) {A → B, C → AB} → {C → B}

{C → AB} → {C → A, C → B}

Proof by Decomposition Rule. In this case {A → B} does not affect the outcome.

b) {KL → M, L → N} → {KN → M}

{K → M, L → M, L → N} // by Decomposition Rule

{K → M, L → MN} // by Combining Rule

Cannot determine if {KN → M} from given FDs, since {N} does not have any implications.

c) {A → C, BD → A, C → D} → {AB → CD}

{A → C, A → D, BD → A} // by Transitivity Rule

{A → C, A → D, B → A, D → A} // by Decomposition Rule

{A → CD, B → A, D → A} // by Union Rule

{A → CD, B → CD, D → A} // by Transitivity Rule

{AB → CD, D → A} // by Union Rule

Proof that the claim is true, the set of original FDs imply that {AB → CD}

d) A relation schema with only 3 attributes is in necessarily at least in 3NF

Then R is 3NF if either:

1. {A, B, C} are trivial FDs. (Condition 1 of 3NF)

2. A combination of FDs involving {A, B, C} implies that all members of the set of FDs that are in the form {X → A} imply that either X is a superkey or A is a prime attribute (Conditions 2 and 3 of 3NF).

Counter Example:

Given FDs {A → C}, we can see that {AB} is the key of the relation. A and B must be prime attributes (as they not on the right side of any relation). This means that the FD {A → C} does not satisfy Condition 1, {A → C} is a non-trivial FD. It also does not satisfy Conditions 2 and 3, since {A} is not the superkey and {C} is not a prime attribute.

In order for a schema with only 3 attributes to be necessarily in 3NF, it must have either only trivial FDs, or at least 2 FDs (in order to satisfy Condition 2 or 3).

2.

Let R = {B, N, S, T, A, R, C}

Let F = {AB → T, A → B, R → C, NS → BT}

a) The candidate keys for this relation are {N, S, A, R}

{N, S, A, R} do not appear on the right of any FD in the set F.

B is not a candidate key as it is on the right of the FD {A → B}.

T is not a candidate key as it appears on the right of the FDs {AB → T, NS → BT}.

C is not a candidate key as it appears on the right of the FD {R → C}.

b) Find the canonical cover of F.

1. Split F into simple FD (minimize right side of every FD).

{AB → T, A → B, R → C, NS → BT} →

{AB → T, A → B, R → C, NS → B, NS → T}

2. Minimize left hand side of every FD.

Let G be the set of minimal FDs that are a cover for F.

{AB → T} → {A → T, B → T} but {A → B, A → B} → {A → T} meaning {A → T} is not necessary. {AB → T} can be minimized to {B → T}.

G = {A → B, B → T, R → C, NS → B, NS → T}

{NS → B} → {N → B , S → B} and {NS → T} → {N → T, S → T} but

{N → B, B → T} and {S → B, B → T} meaning {N → T, S → T} are not necessary. {NS → T} is not necessary for F.

G = {A → B, B → T, R → C, N → B, S → B} is the canonical cover of F

c) A set of FDs does not have a single unique canonical cover; as the order in which the elements are reduced can affect the set of remaining FDs. If the original set is complex enough, different results could be achieved through reduction with the Axioms.

d) R = {B, N, S, T, A, R, C}

F = {AB → T, A → B, R → C, NS → BT}, which has the canonical cover:

G = {A → B, B → T, R → C, N → B, S → B}

The superkey of R is {NSAR}.

R can be decomposed into:

R1 = {BTA} with F1 = {A → BT},

R2 = {NSBT} with F2 = {NS → BT} and

R3 = {RC} with F3 = {R → C}.

Since the remaining key is NSAR, and there is no table with this value, a new table R4 = {NSAR} with trivial FD = {NSAR → NSAR} must be created.

This decomposition is lossless because R1 and R4 have intersection A, which is the key of R1. R2 and R4 have intersection NS, which is the key of R2. R3 and R4 have intersection R, which is the key of R3­.

4.

Let R = {A, B, C, D, E, F}

Let F = {A → BC, BC → A, BD → E, E → F, CF → B}

a) Decomposition of R into  
 R1 = {B, C, D, E}

R2­ = {A, B, C, F}

F1 = {BD → E}

F2 = {A → BC, BC → A, CF → B}

{E → F} missing FD → Not relation preserving.

Intersection of R1 and R2 is BC, which is a not a key of R1 or R2, meaning that the decomposition is lossy.

b) Let F be simplified into:

G = {A → BC, BC → A, BD → E, CF → B, CE → B (by pseudotransitivity of E → F and CF → B)}

Prove BCNF of R1 and R2

R1 = {BCDE} with F1 = {BD → E, CE → B}

R2 = {ABCF} with F2­ = {A → BC, BC → A, CF → B}

R1 can be decomposed into:

R11 = {BDE} with F11 = {BD → E} which is BCNF by condition {X is a superkey}

R12 = {BDC} with F12 = {BDC → BDC} which is BCNF by condition {F is trivial}

R2 can be decomposed into:

R21 = {ABC} with F21 = {A → BC} which is BCNF by condition {X is a superkey}

R22 = {AF} with F22 = {AF → AF} which is BCNF by condition {F is trivial}

The decomposition trees for R1 and R2 are as follows:

R1 = {BCDE}

F1 = {BD → E, CE → B}

R11 = {BDE}

F11 = {BD → E}

R12 = {CDE}

F12 = {BDC → BDC}

R11 ∩ R12 = BD

R2 = {ABCF}

F2 = {A → BC, BC → A, CF → F}

R21 = {ABC}

F21 = {A → BC}

R22 = {AF}

F22 = {AF → AF}

R11 ∩ R22 = A

c) R3 = {CEF} would use F3 = {E → F}.

This cannot be in BCNF as E is not the superkey (which is {CE}) and F3 is not trivial. It also cannot be in 3NF because {F} is not part of the superkey.